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Very long time scales and black hole thermal equilibrium

José L. F. Barbón*
Theory Division, CERN
CH-1211 Geneva 23, Switzerland
E-mail: barbon@cern.ch

Eliezer Rabinovici
Racah Institute of Physics, The Hebrew University
Jerusalem 91904, Israel
E-mail: eliezer@vms.huji.ac.il

Abstract: We estimate the very long time behaviour of correlation functions in the presence of eternal black holes. It was pointed out by Maldacena [2] that their vanishing would lead to a violation of a unitarity-based bound. The value of the bound is obtained from the holographic dual field theory. The correlators indeed vanish in a semiclassical bulk approximation. We trace the origin of their vanishing to the continuum energy spectrum in the presence of event horizons. We elaborate on the two very long time scales involved: one associated with the black hole and the other with a thermal gas in the vacuum background. We find that assigning a role to the thermal gas background, as suggested in the above work, does restore the compliance with a time-averaged unitarity bound. We also find that additional configurations are needed to explain the expected time dependence of the Poincaré recurrences and their magnitude. It is suggested that, while a semiclassical black hole does reproduce faithfully "coarse grained" properties of the system, additional dynamical features of the horizon may be necessary to resolve a finer grained information-loss problem. In particular, an effectively formed stretched horizon could yield the desired results.

Keywords: 1/N Expansion, AdS-CFT and dS-CFT Correspondence, Black Holes.

*On leave from Departamento de Física de Partículas da Universidade de Santiago de Compostela, Spain.

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1. Introduction

String theory seems a formidable multi-faceted fortress of consistency although it shows a less determined side when needing to choose a vacuum or face Nature. As large issues have failed so far to breech its walls perhaps the nitty-gritty picking of small details will suggest where it needs to be modified. A context to study such important fine details is the question of possible information loss in black holes. An aspect of this problem is the search for a more complete understanding of the propagation of strings on backgrounds with spacelike singularities. In fact one may need to await a nonperturbative formulation of string theory to resolve it. The AdS/CFT correspondence offers such a formulation for some string backgrounds [1]. In particular it was found that field theories with a discrete energy spectrum are dual to strings propagating on mixed backgrounds sharing the same conformal boundary. For example, for the case of an asymptotic $\mathrm{AdS}_5 \times \mathbf{S}^5$ both thermal AdS and a black hole in AdS need to be considered above a known temperature. The value of thermodynamical quantities such as the entropy are reproduced and accounted for by the quarks and gluons of an $\mathcal{N} = 4$ supersymmetric CFT. This nonperturbative result gives, for the appropriate temperature, a classical black hole entropy: a standard field theory result encodes a less standard thermodynamical result in gravity.

These thermodynamical quantities measure gross features of the system such as the total number of states. One would like to probe the details further and find to what extent the low-energy supergravity "master fields" arising in the large-N limit of the correspondence are sensitive also to the finer structure of the system such as its energy level spacing. For example, on the nonperturbative field-theory side, the infrared cutoff imposed by the finite radius sphere $\mathbf{S}^3$ enforces a discrete energy spectrum. The bulk realization of this is less clear, yet, these finer details can be related to the study of the possible unitary evolution of black holes. Nonperturbative completions of string theory such as the above mentioned AdS/CFT correspondence leave no room for unitarity violations as a matter of principle, but the detailed manner in which this is enforced in practice is yet to be clarified. Indeed, here we search for the possible need of new dynamical effects.

Recently, a formal criterion for detecting the absence of information loss has been proposed by Maldacena, based on the detailed study of thermal equilibrium between a large eternal black hole and its thermal radiation in AdS spacetime [2]. Since one studies a stationary problem involving possibly very large black holes, one has the impression that there are no obstacles in obtaining definite answers within the semiclassical approximation. However it turns out that this is actually a situation in which the finer details are tested. The suggestion in [2] was to follow the detailed very long time properties of correlation functions. In [3, 4, 5] this program was pursued as a particularly incisive probe of an analogous problem in quantum de Sitter space. It was suggested to examine the manner in which a form of a time-independent lower bound on such correlation functions was respected. Bounds of such nature are known to occur in systems subject to Poincaré recurrences, characteristic of finite bounded systems which evolve in a unitary way, such as the nonperturbative holographic field theory ensures. In general one expects that an initial perturbation of a given thermal system will be damped by the usual thermal dissipation effects, this decay continues as long as the time scale is too short to resolve possible gaps in the spectrum. Eventually for a unitary system with a discrete spectrum the perturbation is prevented from dying out. Moreover for any required precision there will exist a time for which the correlation function returns to its initial value within the required precision.

In [2] it was pointed out that semiclassically, on the bulk side, the correlation function does decay to zero in the presence of a black hole and thus violates the bound. To the contrary, in a thermal AdS background the time-averaged correlation functions do respect a lower non-vanishing bound. The proposal of [2] aims at recovering the recurrences by summing over thermodynamically subleading backgrounds with discrete spectrum, such as the thermal AdS manifold. This has the appealing feature of requiring two spacetimes, with different topology, to contribute in the same string theory (for earlier work in this context see [6, 7, 8].)

In this note we discuss in more detail the nature of this proposal (see [9] for related discussions in the case of three-dimensional black holes). In particular, we show that Poincaré recurrences can indeed be absent for non-unitary systems. i.e. a system with non-coherent quantum evolution [10] will show no recurrences despite having finite entropy. On the other hand, we point out that the absence of Poincaré recurrences in semiclassical correlation functions is tied to the existence of a continuous spectrum of modes in the presence of black hole horizons. This is a property of the "bare" horizon, as described by General Relativity, and could be modified by nonperturbative dynamical effects. Thus the absence of Poincaré recurrences in this case may be an artefact of the semiclassical approximation.

We examine the proposal that recurrences are restored by topological diversity. We present some evidence that, as tempting as the proposed resolution is, it may not recapture some the expected detailed features of the system. While it does lead to a result respecting a lower bound on a time averaged normalized correlation function, it actually seems to do it in a manner reflecting neither the time scale of the recurrences nor the expected amplitude of the correlation function. The time scale it provides is much shorter than could be expected, and the amplitude much smaller than could be expected, although their product does conspire so as to respect the bound.

This paper is organized as follows. Sections 2 and 3 are a review of known facts that serve to focus the problem. In section 2 we review the intuition behind the classical and quantum bounds on correlation functions and their time average. We stress that, for either unbounded or generic non-unitary systems, the correlation functions and their averages vanish. In section 3 we recall the source of the continuous portion of the spectrum of particles in the presence of a black hole and contrast it with the discrete spectrum in the absence of event horizons. In section 4 we evaluate the semiclassical prediction based on the topology change processes suggested in [2]. We estimate the time-averaged lower bounds in two examples: the Schwarzchild black hole in a box and AdS black holes in various dimensions. Some particular behaviour is noted for the three dimensional case.

In a sense we claim that while the semiclassical black hole master field picture is very valuable for computing inclusive properties of the system such as entropy and free energy, the appreciation of finer properties such as the Poincaré recurrences may require finer tools. We end by discussing the possible form of such tools.

2. Unitarity versus Poincaré recurrences

Poincaré recurrences in classical systems are a consequence of the compactness of the energy shell in phase space, together with Liouville's theorem, which states that the canonical flow preserves phase-space volume. It then follows that the time evolution of any finite-volume domain of phase space, Ω, will eventually intersect itself. To see this, consider a series of discrete "snapshots" of the time evolution with fixed time increment Ω, $U(\Omega)$, $U^2(\Omega)$, $\dots$, $U^n(\Omega), \dots$ Since all the images of Ω have the same volume and the phase space is compact, there must be two images in the list with a non-vanishing intersection: $U^k(\Omega) \cap U^l(\Omega) \neq \emptyset$. It follows that $\Omega \cap U^n(\Omega) \neq \emptyset$ for $n = k - l$. This is Poincaré's "eternal return" [11]. There are two independent ways in which we may evade such eternal return. Either we may have access to a *non compact* phase space, or we may consider time flows that do not preserve phase-space volume.

The quantum analogue of Liouville's theorem is the unitarity of the time evolution. The compactness of phase space corresponds to having a finite number of states under the energy shell, i.e. the relevant energy spectrum is *discrete*. The phase-space volume of the initial set Ω is roughly related to the degree of "purity" of the initial quantum state.

A useful quantity to monitor quantum recurrences is the time correlation function of an observable A,

$$G_\rho\,(t) = \mathrm{Tr}\,[\rho\, A(t)\, A(0)]\ , \tag{2.1}$$

where we take $t > 0$ and ρ denotes the density matrix characterizing the initial state of the system. The decay of this correlation function in time gives information about the dissipation of perturbations of typical equilibrium states, such as the canonical thermal state at temperature $1/\beta$:

$$\rho_\beta = \frac{e^{-\beta H}}{Z(\beta)}\,, \tag{2.2}$$

where $Z(\beta) = \mathrm{Tr}\exp(-\beta H)$ is the canonical partition function. A more fundamental equilibrium state for bounded isolated systems is actually the microcanonical state

$$\rho_E = e^{-S(E)}\,\Theta(E-H)\,, \tag{2.3}$$

where Θ is the step function and

$$S(E) = \log \mathrm{Tr}\,\Theta(E-H) \tag{2.4}$$

is the microcanonical entropy. For a bounded system with discrete energy spectrum and unitary evolution,

$$A(t) = e^{itH}\,A(0)\,e^{-itH}\ , \tag{2.5}$$

the thermal dissipation cannot be complete at very long times. For example, the microcanonical correlation function takes the form

$$G_E\,(t) = e^{-S(E)} \sum_{E_i, E_j \leq E} |A_{ij}|^2\, e^{i(E_i - E_j)t}\,. \tag{2.6}$$

In terms of the frequencies $\omega_{ij} = E_i - E_j$, we can define the so-called Heisenberg time (c.f. [12]) by $t_H = 1/\langle\omega\rangle$, where $\langle\omega\rangle$ denotes an average of the frequencies that we may estimate as the total energy divided by the total number of states: $\langle\omega\rangle \sim E\,\exp(-S(E))$. The Heisenberg time is the time scale that reflects the discreteness of the spectrum, i.e. for $t \ll t_H$ we can approximate the spectrum as continuous. In this case, if the matrix elements of A in the energy basis have a frequency width Γ, the correlator will decay with a characteristic lifetime of order Γ^{-1}, i.e. we expect standard dissipative behaviour for times $\Gamma^{-1} \ll t \ll t_H$.

For $t > t_H$ most of the phases in (2.6) would have completed a period and the function $G_E\,(t)$ starts to show large irregular oscillations. In fact, $G_E\,(t)$ as defined by (2.6) is a quasiperiodic function of time; despite thermal damping, it returns arbitrarily close to the initial value over periods of the order of the recurrence time. In order to estimate this recurrence time, let us assume that we have $\mathcal{N}$ different frequencies $\omega_{ij} = E_i - E_j$ that are rationally independent. Then we can picture the phases as a set of $\mathcal{N}$ clocks running at angular velocities ω_{ij} (c.f. [13]). If we demand that they return within a given angular

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accuracy $\Delta\alpha$, the measure of this configuration is $(\Delta\alpha/2\pi)^{\mathcal{N}}$, and the recurrence time is of order $(2\pi/\Delta\alpha)^{\mathcal{N}}/\langle\omega\rangle$, where $\langle\omega\rangle$ is the average frequency. Since $\mathcal{N} \sim \exp(2S)$, the recurrence or Poincaré time scales as a double exponential in the entropy. These scaling properties of the Heisenberg and Poincaré time scales have been explicitly tested in a numerical model in [3].

Notice that setting $\Delta\alpha \sim 2\pi$ we obtain the Heisenberg time scale, corresponding to $\mathcal{O}(1)$ "resurgences" with no emphasis on accuracy, i.e. the Heisenberg time is the smallest possible Poincaré time. If the spectrum shows special regularities the recurrence time can be smaller. For example, all energy levels of a $1+1$ dimensional free quantum field in a circle of radius R are multiples of a single frequency, in which case the correlation function is strictly periodic with period $2\pi R$.

For large systems with positive specific heat the same results should follow for the canonical state ρ_β, in terms of the canonical entropy and internal energy. In this case the energy sums in the analogue of (2.6) are not bounded by the total energy, but only damped by the Boltzman factor $\exp(-\beta E)$. In general, it may be necessary to bound the high-energy off-diagonal matrix elements $|A_{ij}|^2$ in order to ensure convergence of the canonical correlation function $G_\beta\,(t)$, particularly in the limit $t \to 0$. In a local quantum field theory this may require paying due attention to $i\epsilon$ prescriptions, and perhaps working with smeared or regularized operators.

We conclude that time scales associated to large-scale fluctuations or Poincaré recurrences depend exponentially on the entropy. Systems with continuous spectrum have strictly infinite total entropy, and both Heisenberg and Poincaré times are infinite.

2.1 Non-unitary evolution

To further sharpen the relation between recurrences and unitarity, let us consider the loss of quantum coherence envisaged by Hawking in [10]. Time evolutions of this sort violate quantum coherence but preserve hermiticity, positivity and normalization of the density matrix. It was shown in [14] that these systems can be generically modelled by ordinary quantum mechanics coupled to a gaussian random noise.

As a simple example, let us consider a perturbation of the hamiltonian by an operator of the form

$$H \longrightarrow H + C\,J(t)\,, \tag{2.7}$$

where C is a fixed operator that commutes with H but not with the observable A, and $J(t)$ is a gaussian random noise with covariance

$$\langle\, J(t)\, J(t')\,\rangle_J = h\,\delta(t-t')\,. \tag{2.8}$$

We also assume that the operator C is non-degenerate in the energy basis. Then, substituting the formula (2.5) by

$$A(t) = e^{itH + iC\int_0^t dt'\,J(t')}\,A(0)\,e^{-itH - iC\int_0^t dt'\,J(t')} \tag{2.9}$$

we obtain, for the general correlator (2.1),

$$G_\rho\,(t)_J = \sum_{i,j,k} \rho_{ij}\, e^{i(E_j - E_k)\,t}\, A_{jk}\, A_{ki} \left\langle e^{i(C_j - C_k)\int_0^t dt'\,J(t')} \right\rangle_J\,, \tag{2.10}$$

where C_i are the eigenvalues of C in the energy basis. Evaluating the gaussian average one obtains

$$\int \prod_{t'} \left[\frac{dJ(t')}{\sqrt{2\pi}}\right] e^{-\int dt' |J(t')|^2/2h}\, e^{i(C_j - C_k)\int_0^t dt' J(t')} = e^{-h\,|C_j - C_k|^2\, t/2}\,, \tag{2.11}$$

a damping factor that eliminates all possible recurrences as $t \to \infty$. This result implies that Poincaré recurrences may be used as a criterion for unitarity, provided the energy spectrum is truly discrete. On the other hand, one may have a perfectly unitary evolution, and still miss the Poincaré recurrences because the spectrum is continuous.

2.2 Time averages

We now return to unitary time evolutions and define a quantitative measure of the quasi-periodicity of correlators. First we define a normalized correlator

$$L(t) \equiv \left|\frac{G(t)}{G(0)}\right|^2\,, \tag{2.12}$$

which is independent of the overall normalization of the operators. Then we can measure the long time resurgences of $L(t)$ by the value of the time average

$$\overline{L} \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, L(t)\,. \tag{2.13}$$

A rather general estimate of the quantity $\overline{L}$ can be obtained for correlations in the canonical ensemble

$$G_\beta\,(t) = \mathrm{Tr}\,[\,\rho_\beta\, A(t)\, B(0)\,] = \frac{1}{Z(\beta)}\sum_{i,j} e^{-\beta E_i} A_{ij}\, B_{ji}\, e^{i(E_i - E_j)t}\,, \tag{2.14}$$

for general observables A, B with vanishing diagonal matrix elements in the basis of energy eigenvectors. Following [4] we have

$$\overline{L} = \frac{\sum_{i,j,k,l} e^{-\beta(E_i+E_k)}\, A_{ij}\, B^*_{ij}\, A^*_{kl}\, B_{kl}\, \delta_{i-j-k+l}}{\sum_{i,j,k,l} e^{-\beta(E_i+E_k)}\, A_{ij}\, B^*_{ij}\, A^*_{kl}\, B_{kl}}\,. \tag{2.15}$$

The main difference between numerator and denominator is the Kronecker delta that eliminates one of the index sums. Since all terms in the numerator and denominator are essentially identical, we can imagine that the denominator is bigger roughly by a factor of the number of states that effectively contribute, i.e. the thermodynamical degeneracy $\exp(S)$. A more formal argument can be given for large systems, by passing to continuum notation, approximating the sums over discrete energy levels by integrals according to the rule:

$$\sum_n f(E_n) \approx \int dn\, f(E(n)) = \int d\mu(E)\, f(E)\,, \tag{2.16}$$

where

$$d\mu(E) = dE\,\frac{dn(E)}{dE}\,. \tag{2.17}$$

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Using $n(E) = \exp S(E)$ and defining the microcanonical temperature function $\beta(E) = dS(E)/dE$ we have

$$d\mu(E) = dE\, \beta(E)\, e^{S(E)}\,. \tag{2.18}$$

In (2.18), $S(E)$ is a smooth function of the continuous energy variable E. It has the interpretation of an interpolating function that agrees with (2.4) on the discrete energy levels. This *does not* mean that the spectrum is continuous and, in particular, the total entropy $S(E)$ is a finite function.

The time average (2.15) can be approximated as

$$\overline{L} \approx \frac{\int d\mu(E_1)\, d\mu(E_2)\, d\mu(E_3)\, A_{E_1,E_2}\, B^*_{E_1,E_2}\, A^*_{E_3,E'}\, B_{E_3,E'}\, e^{-\beta(E_1+E_3)}}{\int d\mu(E_1)\, d\mu(E_2)\, d\mu(E_3)\, d\mu(E_4)\, A_{E_1,E_2}\, B^*_{E_1,E_2}\, A^*_{E_3,E_4}\, B_{E_3,E_4}\, e^{-\beta(E_1+E_3)}} \tag{2.19}$$

where $E' = E_2 + E_3 - E_1$. We can now evaluate the integrals over E_1 and E_3 in the saddle point approximation. Both saddle points are identical and located at $E = E(\beta) = E_\beta$, the internal energy that solves the equation:

$$\left(\frac{\partial S}{\partial E}\right)_{E=E_\beta} = \beta\,. \tag{2.20}$$

For large systems with positive specific heat $C_V \gg 1$ we may neglect the fluctuations around the saddle point and get

$$\overline{L} \approx \frac{\int dE_2\, \beta(E_2)\, |A(E_\beta,E_2)|^2\, |B(E_\beta,E_2)|^2\, e^{S(E_2)}}{\left|\int dE_2\, \beta(E_2)\, A(E_\beta,E_2)\, B(E_\beta,E_2)^*\, e^{S(E_2)}\right|^2}\,. \tag{2.21}$$

Notice that the factors proportional to the partition function at the saddle point, $\exp(S(\beta) - \beta E(\beta))$, cancel between numerator and denominator. Now let us assume that the operators A and B have matrix elements around $E(\beta)$ consisting of a band of width $\Gamma/2 \ll E(\beta)$ and approximate the integrands by their mean value within this band.[1] Then we obtain the estimate (c.f. [4])

$$\overline{L} \sim \frac{\beta\,\Gamma \cdot e^{S(\beta)}}{\left[\beta\,\Gamma \cdot e^{S(\beta)}\right]^2} \sim \frac{e^{-S(\beta)}}{\beta\,\Gamma}\,, \tag{2.22}$$

up to a ratio of matrix elements corresponding to energies close to E_β. The proportionality factor in (2.22) can also get contributions that depend on the precise definition of entropy function. For example, replacing the step function in (2.20) and (2.4) by a Kronecker delta has the effect of replacing β by E_β in (2.22). Apart from these logarithmic ambiguities in its definition, the entropy factor in the exponent has corrections of $\mathcal{O}(\beta\,\Gamma)$, a quantity that we assume finite in the thermodynamical or semiclassical limits.

Thus, $\overline{L}$ is bounded strictly above zero provided the thermodynamic entropy is finite. This shows that a continuous spectrum (such as that of an unbounded system) implies $\overline{L} = 0$ and no Poincaré recurrences.

[1] In local quantum field theory, this technical requirement may depend on an appropriate regularization of the operators, particularly in the definition of $G_\beta\,(0)$.

One can derive analogous bounds for different types of correlators. One interesting case is the correlation between operator insertions on disjoint thermo-field doubles (c.f. [2]). In this formalism one works with pure states in a doubled Hilbert space, with a precise entanglement between the two copies. Standard thermal correlators correspond to operator insertions on a single copy. Operator insertions on different copies can be interpreted as measurements of slightly non-thermal states. The relevant correlation functions can be obtained by analytic continuation. For example, for the two-point function on different copies,

$$G_\beta\,(t)_{\rm LR} \equiv {\rm Tr}\left[\rho_\beta\, A\left(t-\frac{i\beta}{2}\right) B(0)\right]. \tag{2.23}$$

Aplying similar methods to this correlator[2] we obtain the estimate of the appropriate time average:

$$\overline{L}_{\rm LR} \sim \exp(-K(\beta))\,, \qquad \text{where } K(\beta) = 2\beta\,\left(F(\beta) - F(\beta/2)\right) - S(\beta)\,, \tag{2.24}$$

and $F(\beta) = E(\beta) - S(\beta)/\beta$ denotes the canonical free energy.

3. No recurrences in the presence of event horizons

For a bounded gravitating system, black holes should dominate the thermal ensemble at high energies. In that case we expect the bound (2.22) to hold with β the inverse Hawking temperature and $S(\beta) = A_H/4G_{\rm N}$ the Bekenstein–Hawking entropy. Thus, the expected recurrence index for black holes has a nonperturbative scaling with Newton's constant: $\overline{L} \sim \exp(-1/G_{\rm N})$. This suggests that it might be calculable in the semiclassical approximation.

On the other hand, the infinite redshift of an event horizon implies that a quantum field can accumulate an infinite number of modes there, with a finite cost of energy (c.f. [15]). Hence, perturbative correlation functions should reflect a continuous spectrum of excitations and produce $\overline{L} = 0$. In this section we review this known fact and identify some related subtleties of the euclidean formalism and the procedures of analytic continuation.

3.1 The continuous spectrum

Let us consider a static background metric of the general form

$$ds^2 = -g(r)\,dt^2 + \frac{dr^2}{g(r)} + r^2\,{\rm d}\Omega^2_{d-2}\,, \tag{3.1}$$

where $r = r_0$ is a non-degenerate event horizon, $g(r_0) = 0$, with Hawking temperature $\beta^{-1} = g'(r_0)/4\pi$.

The natural analogue of (2.14) in the background of a black hole is a thermal correlation function of the form

$$\mathcal{G}(t,t') = \frac{{\rm Tr}\,\left[e^{-\beta\mathcal{H}}\,\phi(t,r,\Omega)\,\phi(t',r',\Omega')\right]}{{\rm Tr}\;e^{-\beta\mathcal{H}}}\,, \tag{3.2}$$

[2]One interesting difference with (2.14) is that now all energy sums are damped by Boltzman factors, improving the convergence in the $t\to 0$ limit.

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where $t - t' > 0$ and ϕ is a perturbative field in the background (3.1). In the notation of section 2, this would correspond to $A(t) = \phi(t, r, \Omega)$ and $B(t') = \phi(t', r', \Omega')$. The splitting of spatial coordinates r, Ω avoids ultraviolet divergences in the $t - t' \to 0$ limit, although there remain possible on-shell singularities that must be handled with the appropriate $i\epsilon$ prescription.

The operator $\mathcal{H}$ in (3.2) is the hamiltonian with respect to the asymptotic time variable t, and is defined *perturbatively* around (3.1). In the free approximation for a bosonic field ϕ we have

$$\mathcal{H}_{\text{free}} = \sum_\omega \omega \left(N_\omega + \frac{1}{2} \right), \tag{3.3}$$

where N_ω is the occupation number of the mode with frequency ω. The large-time behaviour of such correlation functions reduces then to the properties of the frequency spectrum ω.

In the free approximation, we can obtain the frequency spectrum by studying the wave equation

$$(\nabla^2 - m^2)\, \phi(x) = 0 \tag{3.4}$$

in a basis of modes with definite frequency

$$\phi(t, r, \Omega) = \frac{1}{\sqrt{2\omega}}\, e^{-i\omega t}\, \phi_\omega\,(r, \Omega)\,, \tag{3.5}$$

and normalized in the Klein–Gordon inner product:

$$\langle\, \psi \,|\, \varphi \,\rangle_{\text{KG}} = i \int_{t=\text{const}} d\sigma^\mu\, \psi^*\, \overleftrightarrow{\partial}_\mu\, \varphi\,. \tag{3.6}$$

Then, the free approximation to the thermal Green's function takes the usual form

$$\mathcal{G}(t, t')_{\text{free}} = \sum_\omega \frac{\phi_\omega(r, \Omega)\, \phi_\omega(r', \Omega')^*}{2\omega} \left[(1 + n_\omega)\, e^{-i\omega\,(t-t')} + n_\omega\, e^{i\omega\,(t-t')} \right], \tag{3.7}$$

with $n_\omega = \left(e^{\beta\omega} - 1 \right)^{-1}$ the average occupation number in the thermal ensemble. Replacing $t \to t - i\beta/2$ in (3.7) we obtain the analogue of (2.23), which is ultraviolet-finite in the $t - t' \to 0$ limit, even for $r = r'$, $\Omega = \Omega'$, and is free from on-shell singularities. However, its long time behaviour is governed by the same frequency spectrum ω.

We can further factor out the $\mathrm{SO}(d-1)$ symmetry by writing

$$\phi_\omega(r, \Omega) = r^{\frac{2-d}{2}}\, f_{\omega,\ell}(r)\, Y_\ell(\Omega)\,, \tag{3.8}$$

with Y_ℓ the appropriate spherical harmonic on $\mathbf{S}^{d-2}$:

$$-\nabla^2_\Omega\, Y_\ell(\Omega) = C_\ell\, Y_\ell(\Omega)\,.$$

The frequency and radial wave function are determined by the Schrödinger problem

$$\left[-\frac{d^2}{dr_*^2} + V_{\text{eff}}(r_*) \right] f_{\omega,\ell}(r_*) = \omega^2\, f_{\omega,\ell}(r_*)\,, \tag{3.9}$$

with

$$V_{\text{eff}} = \frac{d-2}{2}\, g(r) \left(\frac{g'(r)}{r} + \frac{d-4}{2r^2}\, g(r) \right) + g(r) \left(\frac{C_\ell}{r^2} + m^2 \right) . \tag{3.10}$$

Here we have defined the Regge–Wheeler or "tortoise" coordinate $dr_* = dr/g(r)$. In terms of the r_* coordinate, the eigenvalue problem (3.9) inherits a standard L^2 inner product from (3.6).

For asymptotically minkowskian spacetimes, $g(r) \to 1$ as $r \to \infty$, the effective potential asymptotes to $V_{\text{eff}} \to m^2$, and we obtain the standard continuum spectrum of frequencies for $\omega > m$. Near a non-degenerate horizon, $g(r_0) = 0$, the effective potential shows the universal scaling

$$V_{\text{eff}}(r_*) \propto e^{4\pi r_*/\beta} , \qquad \text{as} \qquad r_* \to -\infty , \tag{3.11}$$

with $1/\beta = g'(r_0)/4\pi$ the Hawking temperature. We see that the spectrum of *positive real* frequencies will be continuous, quite independently of the properties of the potential for $r \gg r_0$. In particular, enclosing the system in a large box by placing a reflecting wall at large and finite r fails to discretize the spectrum, due to the exponential tail (3.11).

It is instructive here to consider the large r Anti-de Sitter asymptotics $g(r) \to r^2/R^2$ from the point of view of the effective Schrödinger problem in tortoise coordinates. For $r \to \infty$ the tortoise coordinate approaches a maximum value $(r_*)_{\text{max}} = \pi R/2$. In addition, the effective potential diverges as $r_* \to \pi R/2$. So, we recover the known fact that AdS spaces behave effectively as finite-volume cavities. For vacuum AdS in global coordinates, $g(r) = 1 + r^2/R^2$, the tortoise coordinate near the origin is $r_* \sim r$, so that the range of the effective potential is finite: $r_* \in [0, \pi R/2]$. The spectrum is thus discrete in pure AdS, and continuous in the AdS black hole. However, the Poincaré patch of vacuum AdS, with $g(r) = r^2/R^2$ has continuous spectrum, as $r = 0$ is a degenerate horizon with $g(0) = g'(0) = 0$.

The frequency spectrum in the presence of horizons is thus continuous in the free approximation. Perturbative corrections in the black hole background are constructed in terms of the free Green's functions, and will not discretize the spectrum (unless perturbation theory itself breaks down.) Hence, we have perturbative correlation functions $\mathcal{G}(t)$ with infinite Heisenberg time. Defining the corresponding time average in the background of interest,

$$\overline{\mathcal{L}} \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \left| \frac{\mathcal{G}(t)}{\mathcal{G}(0)} \right|^2 , \tag{3.12}$$

we will find $\overline{\mathcal{L}} = 0$ in the presence of static horizons, at least within the perturbative formalism sketched here. Since $t - t' > 0$ in our Green's functions, they are actually retarded correlators, and the large-time decay can be understood in terms of complex frequencies, a thermal analog of the so-called "quasinormal modes" [9].

3.2 Euclidean formalism

The vanishing of the recurrence index, $\overline{L} = 0$, in the previous discussion is tied to the continuous spectrum of the perturbative hamiltonian $\mathcal{H} = i\,\partial/\partial t$. In the same approximation, the perturbative partition function $\text{Tr}\exp(-\beta\mathcal{H})$ that appears in the normalization of (3.2) is divergent. This means that the defining expression (3.2) is somewhat formal (although (3.7) does make sense even for continuous spectrum.) A more rigorous definition of thermal correlation functions is by analytic continuation from euclidean correlation functions. This formalism also offers a vantage point of view on the issues of regularization and renormalization of ultraviolet divergences.

In the semiclassical approach to thermal effects in quantum gravity, one starts from smooth euclidean backgrounds with appropriate boundary conditions, c.f. [16]. For the case of black holes in canonical ensembles, one encounters euclidean manifolds X, with metrics of the form

$$ds^2 = g(r)\,d\tau^2 + \frac{dr^2}{g(r)} + r^2\,d\Omega_{d-2}^2\,, \tag{3.13}$$

with $\tau \equiv \tau + \beta$ and β the inverse Hawking temperature. With this periodicity of the euclidean time τ the metric (3.13) is smooth at the horizon, defined by $g(r_0) = 0$, and[3] is restricted to the region $r \geq r_0$.

The thermal partition function is given by $Z(\beta) = \exp(-I(X))$, where $I(X)$ is the euclidean effective action evaluated on X. As a function of β, it is interpreted as the canonical free energy, and determines all thermodynamic functions, including the correct black-hole entropy [17]:

$$S(X) = (\beta\,\partial_\beta - 1)\,I(X)\,. \tag{3.14}$$

We shall assume that the asymptotic boundary conditions ensure the stability of the canonical ensemble, in the sense that the specific heat is positive: $C_V > 0$. For black hole states this will require working on a finite box, or in AdS space.

In the gravity perturbative expansion, the euclidean effective action takes the form

$$I(X) = \sum_{n=0}^{\infty} \lambda^{2n-2}\,I_n(X)\,, \tag{3.15}$$

where the I_n are functions of β and whatever other moduli we can adscribe to X. The effective expansion parameter is given by the string coupling $\lambda = g_s$ or some multiple of Newton's constant $\lambda^2 = G_{\rm N}/\ell_{\rm eff}^{d-2}$ in low-energy descriptions with effective cutoff at length scale $\ell_{\rm eff}$. Notice that the leading term $\lambda^{-2} I_0 = I_{cl}$ is precisely the *classical* approximation to the effective action and is of $\mathcal{O}(1/G_{\rm N})$. Although these manipulations are usually carried out within a low-energy field theory, euclidean backgrounds defining worldsheet conformal field theories are also the starting point of perturbative string theory. Even at a nonperturbative level, the euclidean approach to gravitational thermodynamics has been shown to integrate nicely within the AdS/CFT correspondence [7].

Following [16, 18] we can define real-time properties of the thermal ensemble by standard analytic continuation to lorentzian metrics: $\tau = i\,t$. For instance, euclidean correlation functions of local fields in (3.13) can be used to derive lorentzian counterparts on the different patches of the extended lorentzian manifold, such as (3.2), or the vacuum Green's function on the full Kruskal extension of the Schwarzschild metric.

[3]Everything we say in this section can be generalized to the more general case of metrics $ds^2 = g(r)\,d\tau^2 + f(r)^{-1}\,dr^2 + r^2 ds_K^2$, with $f(r_0) = g(r_0) = 0$ and K a $(d-2)$-dimensional compact manifold.

A formal path integral formula for euclidean Green's functions is

$$\mathcal{G}^E(\tau,\tau') = \frac{\int d\mu(\phi,\ldots)\,\phi(\tau,r,\Omega)\,\phi(\tau',r',\Omega')\,e^{-\frac{1}{2}\int_0^\beta (\partial\phi)^2+m^2\phi^2+\ldots}}{\int d\mu(\phi,\ldots)\,e^{-\frac{1}{2}\int_0^\beta (\partial\phi)^2+m^2\phi^2+\ldots}} \tag{3.16}$$

where the dots represent the contribution of other fields to the measure and the action, together with the perturbative interactions. In principle, (3.16) is defined in a low-energy field theory, but we could imagine that such correlation functions exist in an off-shell definition of string field theory. In this expression, $d\mu(\phi,\ldots)$ is the formal measure over the field ϕ, together with the metric fluctuations and all the other bulk degrees of freedom with canonical thermal boundary conditions.

A more rigorous definition is possible in the context of the AdS/CFT correspondence, where one can define correlators of local CFT operators by taking the $r \to \infty$ limit in the bulk field insertions, with wave-function factors $(r/R)^\Delta$ for fields dual to a local CFT operator of conformal dimension Δ. Correlation functions such as (3.16), in terms of local operators in the bulk, can be interpreted as correlators of spatially nonlocal operators in the CFT. Since our discussion of recurrences in section 2 only requires the operators to be local in time, we may carry our arguments directly in terms of (3.16).

The euclidean time direction in (3.13) is isometric, and $\mathcal{G}^E(\tau,\tau')$ is only a function of $\Delta\tau = \tau - \tau'$. After $\mathcal{G}^E(\tau,\tau')$ has been renormalized on the euclidean manifold X, we can define (3.2) by standard analytic continuation in the time argument, $-i\Delta\tau = \Delta t = t - t' > 0$, a procedure that takes care of the appropriate $i\epsilon$ prescription. From the same Hartle–Hawking Green's function one can also define correlators on different thermo-field doubles, which do not suffer from on-shell singularities, by the prescription (2.23).

This definition makes it somewhat paradoxical that the lorentzian Green's function $\mathcal{G}(t,t')$ should show a continuous spectrum of excitations localized at the horizon, because $r = r_0$ is a perfectly regular point of the manifold X. The euclidean Green's function can be written as

$$\mathcal{G}^E(\tau,\tau') = \langle\,\tau\,|\,(K_X)^{-1}\,|\,\tau'\,\rangle + \cdots\,, \tag{3.17}$$

where the dots stand for interaction corrections and K_X is the kinetic operator on X. For a free scalar, it is given by $K_X = -\nabla_X^2 + m^2$. The manifold X is smooth at $r = r_0$ and the spectrum of K_X becomes discrete once we impose a large-volume cutoff. Then, how is it possible for (3.7) to have a spectral decomposition with continuous frequency spectrum? The "disease" of the continuous spectrum has been tied to the phenomenon of divergences in the calculation of black hole entropy [15] and the question of their renormalization [19]. However, here we are obtaining a consequence of this "disease" at the level of a manifestly ultraviolet-finite quantity, such as the long-time free propagator. In this sense, an *infrared* interpretation of the divergence in [15] seems more natural, along the lines of [20, 21].

This phenomenon can be traced back to a subtlety in the analytic continuation from (3.16) to (3.2), which in turn originates on the peculiar topology of the euclidean black hole manifold X. We sketch here the main points, referring the reader to [21] for more details.

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In order to write a hamiltonian representation with respect to $\mathcal{H} = i\,\partial/\partial t$ one must foliate the spacetime in $t =$ constant hypersurfaces. In the original euclidean manifold X this corresponds to $\tau =$ constant hypersurfaces. However, the euclidean time orbits are contractible on X, and the horizon $r = r_0$ is a singularity of the folliation, because it is invariant under translations of τ. For this reason we can expect some subtle behaviour of the hamiltonian representation (3.2). One way of rendering the folliation non-singular is to remove the submanifold at the point $r = r_0$ and define the correlation functions by continuity. Then the manifold $X' \equiv X - \{r = r_0\}$ has a smooth time folliation and differs from X by a set of measure zero. However, it has different topology from X and the "boundary" at $r = r_0$ is non-compact.

We can now consider the operator

$$\widetilde{K}_{X'} = |g_{00}|\, K_X\,, \tag{3.18}$$

which is well defined on X' and has a very simple time dependence. For the metrics of the form (3.13),

$$\widetilde{K}_{X'} = -\partial_\tau^2 - g(r)\, r^{2-d}\, \partial_r\, r^{d-2}\, \partial_r - g(r)\, \nabla_\Omega^2 + m^2\,. \tag{3.19}$$

Instead of the standard covariant inner product

$$\langle\, \psi \,|\, \varphi\, \rangle = \int_X d^d x\, \sqrt{|g|}\, \psi^*(x)\, \varphi(x) \tag{3.20}$$

we may define a rescaled inner product on X':

$$\langle\!\langle\, \psi \,|\, \varphi\, \rangle\!\rangle = \int_{X'} d^d x\, \mu(x)\, \psi^*(x)\, \varphi(x)\,, \qquad \text{where } \mu(x) \equiv \frac{\sqrt{|g|}}{|g_{00}|}\,. \tag{3.21}$$

Then, the Green's function

$$\widetilde{\mathcal{G}}^E(\tau,\tau') = \langle\!\langle\, \tau \,|\, (\widetilde{K}_{X'})^{-1} \,|\, \tau'\, \rangle\!\rangle \tag{3.22}$$

satisfies the same differential equation as the Hartle–Hawking Green's function (3.17), when restricted to X'. On the other hand, the eigenvalue problem for the operator $\widetilde{K}_{X'}$ is given by

$$\widetilde{K}_{X'}\, \widetilde{\psi}_{n,\omega}(\tau, r, \Omega) = \lambda_{n,\omega}\, \widetilde{\psi}_{n,\omega}(\tau, r, \Omega)\,, \tag{3.23}$$

with

$$\lambda_{n,\omega} = \frac{4\pi^2 n^2}{\beta^2} + \omega^2\,, \qquad \widetilde{\psi}_{n,\omega}(\tau, r, \Omega) = \frac{1}{\sqrt{\beta}} e^{2\pi i \tau/\beta}\, \phi_\omega(r,\Omega)\,, \tag{3.24}$$

where $n \in \mathbb{Z}$ and ω^2 solves (3.9). Therefore, on X' we can write

$$\widetilde{\mathcal{G}}^E(\tau,\tau') = \frac{1}{\beta} \sum_n \sum_\omega \frac{e^{2\pi i n(\tau-\tau')/\beta}\, \phi_\omega(r,\Omega)\, \phi_\omega^*(r',\Omega')}{\omega^2 + 4\pi^2 n^2/\beta^2}\,. \tag{3.25}$$

Evaluating now the sum over n using the identity

$$\frac{1}{\beta} \sum_{n\in\mathbb{Z}} \frac{e^{2\pi i n \Delta\tau/\beta}}{\frac{4\pi^2 n^2}{\beta^2} + \omega^2} = \frac{1}{2\omega} \frac{\cosh \omega(\Delta\tau - \beta/2)}{\sinh(\beta\omega/2)} \tag{3.26}$$

and performing the analytic continuation $\Delta\tau = i\,\Delta t$ we obtain (3.7).

Since (3.17) and (3.25) satisfy the same differential equation, we expect that $\widetilde{\mathcal{G}}^E$ gives exactly the Hartle–Hawking Green's function on X, when extended by continuity from X'. This last step is not completely free of ambiguities, since the operator $\widetilde{K}_{X'}$ itself is not well defined at $r = r_0$. However, this question can be settled by an explicit calculation in the near-horizon approximation.

3.3 The near-horizon limit

The emergence of continuous spectrum out of smooth euclidean Green's functions can be illustrated with the explicit example of perturbations in the near-horizon region. For $r \approx r_0$ we can approximate the metric by Rindler space, with a pure exponential potential in (3.11). Setting $\xi = 2\sqrt{(r - r_0)/g'(r_0)}$ and $\theta = 2\pi\tau/\beta$, the euclidean Rindler space in the vicinity of the horizon $r \approx r_0$ becomes a copy of flat $\mathbb{R}^d$ space:

$$ds^2 \approx \xi^2 \, d\theta^2 + d\xi^2 + d\mathbf{y}^{\,2} \,. \tag{3.27}$$

In lorentzian signature, the wave functions on the exponential potential are given by Bessel functions

$$\phi_{\mathbf{p},\,\omega}(\xi, \mathbf{y}) = \frac{e^{i\mathbf{p}\cdot\mathbf{y}}}{\pi\,(2\pi)^{\frac{d-2}{2}}} \sqrt{2\nu\,\sinh(\pi\nu)}\, K_{i\nu}(\mu\,\xi) \tag{3.28}$$

where $\mu^2 = m^2 + \mathbf{p}^2$ and $\nu = \beta\omega/2\pi$. Starting from (3.25) we have the explicit representation as a continuous frequency integral

$$\widetilde{\mathcal{G}}^E(\Delta\tau) = \int \frac{d\mathbf{p}}{(2\pi)^{d-2}} e^{i\mathbf{p}\cdot\Delta\mathbf{y}} \int_0^\infty \frac{d\nu}{\pi^2} \cosh[\nu(\Delta\theta - \pi)]\, K_{i\nu}(\mu\,\xi)\, K_{i\nu}(\mu\,\xi')\,. \tag{3.29}$$

The integrals can now be evaluated (see for example appendix A of [21]) to obtain

$$\mathcal{G}^E(\Delta\tau) = \frac{1}{2\pi}\left(\frac{m}{2\pi\Delta s}\right)^{\frac{d-2}{2}} K_{\frac{d-2}{2}}(m\Delta s)\,, \tag{3.30}$$

where Δs is the geodesic distance on $\mathbb{R}^d$. The final result is the Green's function of the laplacian on $\mathbb{R}^d$, i.e. the Hartle–Hawking euclidean Green's function. Thus, we have shown by explicit calculation that a real-time thermal correlator with continuous spectrum at the horizon is the analytic continuation of an euclidean correlator that is perfectly smooth in the vicinity of $r = r_0$. We can now continue (3.30) back to lorentzian signature and check the large time asymptotics of the correlator. Consider the geodesic distance between two points in (3.27) with equal values of ξ and $\mathbf{y}$ and $\Delta\tau < \beta$. We have $\Delta s = 2\xi\sin(\pi\Delta\tau/\beta)$. Performing the analytic continuation $\Delta s \to -2i\,\xi\sinh(\pi\Delta t/\beta)$. Hence, at large Δt the real-time correlator vanishes exponentially and shows no Poincaré recurrences.

Because of the exponential barrier at large r_*, the dissipation of perturbations located at $r \approx r_0$ occurs mostly towards the horizon, with a lifetime given by

$$|\,\mathrm{Im}\,\omega|^{-1} = \frac{2\beta}{\pi(d-1)} = \frac{8}{(d-1)g'(r_0)}\,, \tag{3.31}$$

in the massive case, and

$$|\,\mathrm{Im}\,\omega|^{-1} = \frac{2\beta}{\pi(d-2)} = \frac{8}{(d-2)g'(r_0)}\,, \tag{3.32}$$

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in the massless case. We see that the characteristic time scales are independent of the non-zero mass and are controlled by the Hawking temperature of the black hole. It should be noted however that (3.31), (3.32) must be viewed only as order-of-magnitude estimates of the true black hole's damping frequencies. Since the Rindler approximation regards the horizon as flat $\mathbb{R}^d$, results are not accurate on scales of the Schwarzschild radius, which can be of the same order of magnitude as $|\,\mathrm{Im}\,\omega\,|^{-1}$.

3.4 The brick wall

We have traced the absence of recurrences to the peculiar topological properties of the euclidean manifold X. This special topology guarantees the emergence of a classical contribution to the entropy [17]. At the same time, it renders the analytic continuation to real time completely blind to the spacing of black hole energy levels. Hence, the restoration of the recurrences depends on dynamical effects that change the topology of X. This conclusion holds for any static horizon, including de Sitter in the static patch, and implies that the details of unitary time evolution can only be retrieved by new nonperturbative effects.

The recurrences can be restored if we give up the smooth nature of X and change its topology in an *ad hoc* way. For example, we may cut out the manifold at a distance ε from the horizon, imposing a Dirichlet boundary condition on fields, i.e. 't Hooft's brick-wall model [15]. This reflecting boundary condition makes the domain of r_* compact, and the Schrödinger problem (3.9) yields a discrete frequency spectrum.

The euclidean cutoff manifold X_ε has now cylindrical topology and no classical entropy of Hawking–Gibbons type. The leading contribution to the entropy arises at one-loop level and diverges as $S(X_\varepsilon) \sim A_H/\varepsilon^{d-2}$. If this is equated with the Bekenstein–Hawking entropy, we have an *a posteriori* adjustment of the phenomenological parameter, ε. The effective potential (3.11) for low energy excitations has now discrete spectrum and the recurrence index $\overline{L} \sim \exp(-S(X_\varepsilon))$ has the right order of magnitude. Of course, such modifications of the boundary conditions are not consistent with the systematics of the semiclassical expansion. Rather, one should find well-defined nonperturbative corrections that impose such effective boundary conditions. From our discussion we know that they should involve topology-changing processes, and some candidate configurations will be described in the next section.

4. Poincaré recurrences and topological diversity

The recurrence index was estimated as $\overline{L} \sim \exp(-S)$ for systems with discrete spectrum. This estimate yields $\overline{L} \sim \exp(-A_H/4G_\mathrm{N})$ for a system whose entropy is dominated by a black hole phase. However, a direct analysis of correlation functions in the background of a black hole yields $\overline{L} = 0$ in perturbation theory. In this respect, the recurrence index $\overline{L}$ seems to behave quite differently from the partition function $Z(\beta)$.

Nonperturbative effects of $\mathcal{O}(e^{-1/G_\mathrm{N}})$ arising in the semiclassical expansion must effectively discretize the energy spectrum in order to capture the phenomenon of the recurrences. It is unlikely that string D-instanton corrections in the background (3.13) would achieve

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this goal, at least within the dilute-gas regime. Large-scale fluctuations of the gravitational background that change the global topology of (3.13) might be strong enough [2]. Consider, for example, tunneling transitions between (3.13) and other backgrounds with discrete spectrum of excitations in perturbation theory. In this case we may obtain a non-vanishing result for the recurrence index $\overline{L}$ from the contribution of these backgrounds with discrete spectrum. In this section we obtain a quantitative estimate of these "instanton effects" and we discuss their physical interpretation.

Let us consider a set of perturbative euclidean backgrounds X_α that share the same thermal boundary conditions. The characteristic example of this in the black-hole applications is the substitution of the black-hole metric by the vacuum metric with a gas of perturbative thermal particles at temperature $1/\beta$. In the AdS/CFT context, they correspond to different large-N master fields of the CFT that contribute at the same temperature. In this situation the total measure $d\mu(\phi, \ldots)$ splits into the different perturbative measures $d\mu_\alpha$ around each background,

$$\int d\mu \to \sum_\alpha e^{-I_{cl}(X_\alpha)} \int d\mu_\alpha \,, \tag{4.1}$$

where we have explicitly included the suppression factor by the *classical* euclidean action of each background. The total euclidean correlator is then given by an average over the different backgrounds. Applying the substitution (4.1) to (3.16) we find

$$G^E(\tau, \tau') = \frac{\sum_\alpha e^{-I(X_\alpha)} \, \mathcal{G}^E_\alpha(\tau, \tau')}{\sum_\alpha e^{-I(X_\alpha)}} \,. \tag{4.2}$$

Here $I(X_\alpha) = I_{cl}\,(X_\alpha) + \mathcal{O}(\lambda^0)$ denotes the perturbative expansion of the effective action around the background manifold X_α.

Real-time response functions, denoted collectively by $G(t)$, are defined through the analytic continuation in the time arguments τ, τ' as before:

$$G(t) = \frac{\sum_\alpha e^{-I(X_\alpha)} \, \mathcal{G}_\alpha(t)}{\sum_\alpha e^{-I(X_\alpha)}} \,. \tag{4.3}$$

This expression applies to thermal correlators either of type (2.14) or of type (2.23), provided the gravitational counterparts $\mathcal{G}_\alpha(t)$ are defined accordingly. From this expression we can calculate the "instanton" approximation to $\overline{L}$ with the result

$$\overline{L}_{\rm inst} = \frac{\sum_{\alpha,\beta} e^{-I(X_\alpha)-I(X_\beta)} \, \mathcal{G}_\alpha(0) \, \mathcal{G}_\beta(0)^* \, \overline{\mathcal{L}}_{\alpha\beta}}{\sum_{\alpha,\beta} e^{-I(X_\alpha)-I(X_\beta)} \, \mathcal{G}_\alpha(0) \, \mathcal{G}_\beta(0)^*} \,, \tag{4.4}$$

where we have defined the time averages of the gravity Green's functions:

$$\overline{\mathcal{L}}_{\alpha\beta} \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \mathcal{L}_{\alpha\beta}\,(t) \,, \qquad \mathcal{L}_{\alpha\beta}\,(t) \equiv \frac{\mathcal{G}_\alpha(t)\, \mathcal{G}_\beta(t)^*}{\mathcal{G}_\alpha(0)\, \mathcal{G}_\beta(0)^*} \,. \tag{4.5}$$

The zero-time Green's functions $\mathcal{G}_\alpha(0)$ are of $\mathcal{O}(1)$ in the semiclassical limit. Hence, the denominator of (4.4) is dominated by the manifold with largest partition function at each temperature. We shall denote this thermodynamically-dominating manifold X. On the other hand, the numerator is determined by the competition between the thermodynamical partition function and the time averages $\overline{\mathcal{L}}_{\alpha\beta}$. At any rate, the time averages vanish identicaly for any black-hole background. Hence, the background that dominates the numerator of (4.4), denoted Y, does not contain horizons. In this way we arrive at the final estimate:

$$\overline{L}_{\rm inst} \approx e^{-2\,\Delta I}\,\left|C_{Y/X}\right|^2 \cdot \overline{\mathcal{L}}_Y\,, \tag{4.6}$$

where we denote

$$\Delta I = I(Y) - I(X) \qquad \text{and} \qquad C_{Y/X} = \frac{\mathcal{G}_Y(0)}{\mathcal{G}_X(0)}\,.$$

The value of $\overline{\mathcal{L}}_Y$ is controlled by the eigenvalue spacing of $\mathcal{H}(Y)$, the perturbative hamiltonian of gravitational fluctuations in the background Y. This background has no horizons and $\mathcal{H}(Y)$ has discrete spectrum. Applying the general discussion of section 2 we obtain $\overline{\mathcal{L}}_Y \sim \exp(-S(Y))$. Hence, the instanton approximation to $\overline{L}$ agrees exactly with the bound (2.22) when $X = Y$, i.e. when the thermodynamical free energy is dominated by a thermal gas with no black holes.

At sufficiently large temperatures black holes will be dominating and $X \neq Y$. In this case, the leading exponential suppression of $\overline{L}_{\rm inst}$ is of order $\exp(-2\Delta I) \sim \exp(-1/G_{\rm N})$, and the result of (2.22) is recovered in order of magnitude. The agreement is exact in the strict $G_{\rm N} \to 0$ limit (or $N \to \infty$ in the dual CFT), where the entropy is no more finite and the bound itself vanishes.

We may ask whether the agreement between (2.22) and (4.6) is exact also at the level of $\mathcal{O}(\exp(-1/G_{\rm N}))$ accuracy in the exponential suppression. In order to parametrize the success of the instanton approximation to $\overline{L}$ we define the relative error of the exponent

$$\eta \equiv \frac{S(X) - 2\Delta I}{S(X)}\,. \tag{4.7}$$

The instanton approximation would be succesful if this quantity turned out to be zero in the leading $\mathcal{O}(1/G_{\rm N})$ approximation. We now check the value of η with an explicit computation in two characteristic examples.

4.1 Schwarzschild black holes in a box

The simplest system of a black hole in finite volume is that of a Schwarzschild solution with reflecting boundary conditions on a sphere of radius R. It has the problem that the mathematical characterization of the box is very unphysical. In addition, it lacks a known holographic dual. However, we can assume that such a quantum mechanical description exists and then we can calculate η in the semiclassical approximation, following York, [22].

Let us consider the canonical equilibrium of Schwarzschild black holes with euclidean metric

$$ds^2 = \left(1 - \left(\frac{r_0}{r}\right)^{d-3}\right) d\tau^2 + \frac{dr^2}{(1-(r_0/r)^{d-3})} + r^2\, d\Omega_{d-2}^2 \tag{4.8}$$

in spacetime dimensions $d \geq 4$ with a spherical boundary at $r = R$ kept at fixed temperature $1/\beta$. The period of τ is fixed by the requirement of smoothness at $r = r_0$ and is given by $\tau \equiv \tau + \beta_\tau$, where

$$\beta_\tau = \frac{4\pi}{d-3}\, r_0\,. \tag{4.9}$$

The inverse local temperature at the boundary β is the proper size of the thermal circle at $r = R$, i.e.

$$\beta = \beta_\tau \sqrt{1 - \left(\frac{r_0}{R}\right)^{d-3}} = \frac{4\pi r_0}{d-3} \sqrt{1 - \left(\frac{r_0}{R}\right)^{d-3}}\,. \tag{4.10}$$

This expression shows that such black holes have positive specific heat, $d\beta/dr_0 < 0$, for

$$r_0 > R \left(\frac{2}{d-1}\right)^{\frac{1}{d-3}}\,,$$

and that there is a minimum temperature for equilibrium in the box. Using now the standard expressions for the ADM mass and Bekenstein–Hawking entropy:

$$M = \frac{(d-2)\,\mathrm{vol}\,(\mathbf{S}^{d-2})}{16\pi G_{\mathrm{N}}}\, r_0^{d-3}\,, \qquad S = \frac{\mathrm{vol}\,(\mathbf{S}^{d-2})}{4G_{\mathrm{N}}}\, r_0^{d-2}\,, \tag{4.11}$$

we can obtain the free energy $I = \beta\, M - S$, or

$$I_{\mathrm{bh}} = \frac{\mathrm{vol}\,(\mathbf{S}^{d-2})}{4G_{\mathrm{N}}}\, r_0^{d-2} \left[\frac{d-2}{d-3}\sqrt{1 - \left(\frac{r_0}{R}\right)^{d-3}} - 1\right]. \tag{4.12}$$

With this normalization, the free energy of a pure radiation ball is $I = 0$ to order $\mathcal{O}(1/G_{\mathrm{N}})$. The black hole in equilibrium with radiation dominates over the pure radiation state for Schwarzschild radii above the critical nucleation radius

$$r_0 > r_{\mathrm{nucl}} = R \left(\frac{2d-5}{(d-2)^2}\right)^{\frac{1}{d-3}}\,. \tag{4.13}$$

In this example, the background X is given by a black hole for temperatures above the nucleation temperature. The background Y continues to be the flat-space box. Here $\Delta I = -I_{\mathrm{bh}}$ and the mismatch parameter,

$$\eta = -1 + \frac{2d-4}{d-3}\sqrt{1 - \left(\frac{r_0}{R}\right)^{d-3}}\,, \tag{4.14}$$

varies between $\eta = 1$ at the nucleation temperature, down to $\eta = -1$ at infinite temperature. We see that the instanton evaluation of $\overline{L}$ is generically different from the direct quantum mechanical estimate. At the particular Schwarschild radius

$$r_0 = R \left(\frac{(3d-7)(d-1)}{4(d-2)^2}\right)^{\frac{1}{d-3}} \tag{4.15}$$

we have $\eta = 0$.

4.2 AdS black holes

A more rigorous example in which the boundary is fully specified by a gravitational interaction is the case of black holes in AdS space. This is more interesting, because we may regard the correlators $G(t)$ as defined non-perturbatively in the dual CFT theory on the boundary. In this case, for local operators in the bulk of AdS, the corresponding operators in the CFT are non-local. This is not a problem, since the general considerations of section 2 make no assumptions about the spatial locality of the operators A and B.

Recall the computation of the free-energy difference between the large AdS_d black hole, which we denote X, and the AdS vacuum, denoted Y, in the classical gravity approximation (c.f. [7]),

$$\Delta I = I(Y) - I(X) = \frac{\mathrm{vol}(\mathbf{S}^{d-2})}{4\,G_{\mathrm{N}}} \frac{r_0^d - R^2 r_0^{d-2}}{(d-1)\,r_0^2 + (d-3)R^2}\,, \tag{4.16}$$

where r_0 stands for the horizon radius, related to the inverse temperature by the formula

$$\beta = \frac{4\pi R^2 r_0}{(d-1)\,r_0^2 + (d-3)R^2}\,. \tag{4.17}$$

The mismatch of the instanton method is now given by

$$\eta = \frac{(d-3)r_0^2 + (d-1)\,R^2}{(d-1)\,r_0^2 + (d-3)R^2}\,. \tag{4.18}$$

We find that the disagreement is in general of $\mathcal{O}(1)$. The mismatch between both estimates is maximal at the Hawking–Page phase transition $r_0 = R$ where the ratio $\eta = 1$. At very large temperatures, $\beta \ll R$, it decreases according to the law

$$\eta = \frac{d-3}{d-1} + \frac{1}{2\pi^2(d-1)}\left(\frac{\beta}{R}\right)^2 + \mathcal{O}\left(\frac{\beta^4}{R^4}\right). \tag{4.19}$$

For $d = 3$ this asymptotic formula is exact, without any $\mathcal{O}(\beta^4/R^4)$ corrections. We see that the two estimates of $\overline{L}$ are exponentially different at any finite temperature above the phase transition. They only agree for $d = 3$ in the strictly infinite temperature limit.

In $d = 3$ there is a rich structure of black holes related by $\mathrm{SL}(2,\mathbb{Z})$ transformations [23]. However, the previous analysis still applies, provided we only consider non-rotating black holes. At zero angular momentum, all the extra black holes are subleading, either to the vacuum AdS at low temperatures, or to the high-temperature black hole beyond the phase transition.

The $d = 3$ case presents however an interesting peculiarity. Unlike the $d > 3$ case, BTZ black holes do not approach the vacuum in the zero-mass limit. The smallest regular black hole has zero mass in units in which the energy of the vacuum is the Casimir energy of the Neveu–Schwarz sector, namely

$$E_{\mathrm{NS\,vac}} = -\frac{C}{12R} \tag{4.20}$$

with C the CFT's central charge. The zero-mass black hole in the NS sector is degenerate with the Ramond vacuum and has zero temperature and metric

$$ds^2 = \frac{r^2}{R^2}\,(-dt^2 + R^2\,d\varphi^2) + R^2\frac{dr^2}{r^2}\,. \tag{4.21}$$

Thus, this metric describes the Ramond vacuum if we assign periodic boundary conditions to the fermions in the asymptotic spatial cycle. But this metric is just the Poincaré patch of the vacuum AdS manifold, so $r = 0$ is a horizon, with the associated problem of the continuous spectrum of modes [24]. In addition, this horizon is actually singular on account of the compact nature of the angle φ. So, the Ramond sector of the two-dimensional CFT is an example where the vacuum manifold has already the disease of the continuous spectrum,[4] and it seems impossible that large-distance semiclassical effects could restore the Poincaré recurrences.

A similar phenomenon takes place in AdS spaces of arbitrary dimension at sufficiently large temperatures. At very small β, the vacuum AdS$_d$ manifold with a thermal gas becomes unstable, either by the Jeans instability or by other stringy effects. In such a situation, we would lack an appropriate Y background to recover a nonvanishing $\overline{L}$. For example, the standard AdS$_5 \times \mathbf{S}^5$ model develops thermal tachyons when the temperature reaches either the Hagedorn temperature in the bulk $R/\beta \sim (g_s N)^{1/4}$, or the Jeans temperature $R/\beta \sim N^{1/5}$ (c.f. [8, 26, 27]).[5]

4.3 Long and very long time scales

We have seen that, while the instanton approximation (4.6) to the unitarity bound (2.22) is right in order of magnitude, it is unlikely to be exact to order $\exp(-1/\lambda^2)$. We can argue this by noticing that $\overline{L}_{\rm inst} \sim \exp(-2\Delta I)$ is universal for all types of two-point functions that can be obtained as analytic continuation of the euclidean Hartle–Hawking Green's functions. In particular, it applies to the $\overline{L}_{\rm LR}$ average in the thermo-field double. However, we know that the direct bounds of section 2 have a slight dependence on the class of correlation function considered (compare (2.22) with (2.24)). Furthermore, the evaluation of $\overline{L}_{\rm inst}$ relies on the existence of an appropriate Y background with discrete spectrum, a requirement that is not met in the RR sector of BTZ black holes, or even in general AdS spaces for sufficiently large temperatures.

In fact, despite the agreement between (2.22) and (4.6), the instanton processes considered so far do not directly address the question of Poincaré recurrences. Let us consider the instanton prediction for the very long time behaviour of the normalized ratio $L(t)$, rather than the time average. Since $\lim_{t\to\infty} \mathcal{G}_X(t) = 0$, we have

$$L(t)_{\rm inst} \approx e^{-2\Delta I} \left|C_{Y/X}\right|^2 \cdot \mathcal{L}_Y(t)\,, \tag{4.22}$$

for sufficiently large times.

In (4.22) the period of quasiperiodicity of $L(t)_{\rm inst}$ is that of $\mathcal{L}_Y(t)$. This is controlled by the average eigenvalue spacing of the perturbative hamiltonian $\mathcal{H}(Y)$, which is of $\mathcal{O}(1)$ in the semiclassical expansion in powers of λ. Therefore, the Heisenberg time of $L(t)_{\rm inst}$ scales as $t_H(Y) = \mathcal{O}(1)$. On the other hand, the exact CFT has a dense band of states with

[4] In this case one can hope for improvements of the semiclassical approximation along the lines of [25].

[5] This is just as well, because otherwise the background Y would dominate the thermodynamics at temperatures $R/\beta \gg N^{1/3}$, having a non-holographic scaling in the temperature $I_Y \sim -(R/\beta)^9$, to be compared with $I_X \sim -N^2\,(R/\beta)^3$.

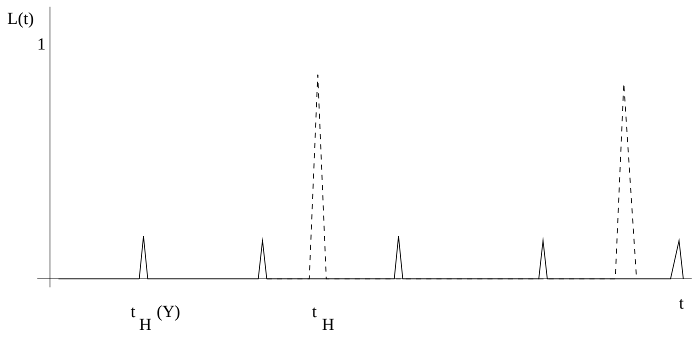

Figure 1: Schematic representation of the very long time behaviour of $L(t)_{\rm inst}$ (dark line) compared to the expected pattern for the exact quantity $L(t)$. The resurgences of $L(t)_{\rm inst}$ occur with periods of order $t_H(Y) = \mathcal{O}(\lambda^0)$ and have amplitude of order $e^{-1/\lambda^2} \ll 1$. The expectations for the exact CFT, in the dashed line, are $\mathcal{O}(1)$ resurgences with a much larger period $t_H \sim e^{1/\lambda^2} \gg t_H(Y)$, corresponding to tiny energy spacings of order e^{-1/λ^2}. Despite the gross difference of both profiles, the infinite time average is $\mathcal{O}(e^{-1/\lambda^2})$ for both of them.

spacing of order $\exp(-S) \sim \exp(-1/\lambda^2)$, leading to a Heisenberg time of order $t_H \sim e^{1/\lambda^2}$. Since $t_H(Y) \ll t_H$, the single instanton approximation does not reflect the correct time scale of the problem. In addition, while $\mathcal{L}_Y(t)$ does show $\mathcal{O}(1)$ resurgences on periods of order $t_H(Y)$, the overall prefactor in (4.22) means that the resurgences of $L(t)_{\rm inst}$ are only of size $e^{-2\Delta I} \sim e^{-1/\lambda^2} \ll 1$. Thus we conclude that the single instanton approximation to $L(t)$ has the wrong pattern of Poincaré recurrences, both at the level of the period and also the amplitude. Interestingly, these two errors tend to cancel one another when considering the infinite time average $\overline{L}_{\rm inst}$, which simply means that time averages can be misleading in this particular problem.

However, even though the small noise (4.22) is not directly related to the true Poincaré recurrences, it does represent an interesting correction to the semiclassical correlators in the black hole background, showing the first signs of information retrieval. In particular, its effects are significant much before the Poincaré time. To see this we can start from (4.3), before taking the time average, and estimate $L(t)$ as

$$L(t)_{\rm inst} = \mathcal{L}_X\,(t) + e^{-2\Delta I}\left|C_{Y/X}\right|^2 \mathcal{L}_Y\,(t) + 2\,e^{-\Delta I}\,{\rm Re}\,\left[\,C_{Y/X}\,\mathcal{L}_{XY}\,(t)\,\right] + \cdots \tag{4.23}$$

where the amplitude of $\mathcal{L}_{XY}\,(t)$ is of order $|\mathcal{L}_X\,(t)\cdot\mathcal{L}_Y\,(t)|^{1/2}$. Approximating the long time behaviour of the black hole correlation functions as

$$\mathcal{L}_X\,(t) \sim \exp(-2\Gamma\,t)\,, \tag{4.24}$$

we find that the $\mathcal{O}(1)$ "bumps" of $\mathcal{L}_Y\,(t)$ represent a significant correction to $\mathcal{L}_X\,(t)$ for critical times t_c such that $\exp(-\Gamma t_c) \sim \exp(-\Delta I)$, that is

$$t_c \sim \frac{\Delta I}{\Gamma}\,, \tag{4.25}$$

a very large time for macroscopic black holes, but still exponentially smaller than the true

Heisenberg time of the system. In fact, in the strict semiclassical limit, where $\lambda^2 \sim 1/N^2$ is smaller than any other dimensionless quantity in the system, one has $t_c \gg t_H(Y)$, and the "instanton noise" (4.23) looks like a stabilization of the time correlator $L(t)$, when considering time scales of order t_c.

We have seen in section 3 that Γ is proportional to $1/\beta$ up to perturbative corrections, which gives $t_c \sim \beta\, \Delta I$. For very massive fields in AdS_3 black holes, corresponding to CFT_2 operators of very high dimension $\Delta \sim m\, R \gg 1$, the WKB approximation to the correlator yields $\Gamma \sim m\, R/\beta$ and we recover the "fluctuation time" $t_c \sim \beta\, \Delta I/mR$ of the first paper in [28].

5. Discussion

Black holes may violate the Poincaré theorem on recurrences in two different ways. First, there will be no recurrences if the evaporation process violates quantum coherence. However, even if we keep the standard unitary formalism of quantum mechanics, recurrences are averted by the effectively non-compact phase space of the black hole horizon. This is exactly what happens in the semiclassical approximation to equilibrium gravitational thermodynamics. The success of the semiclassical treatment for thermodynamical quantities is largely based on the prescription of Hartle and Hawking, that incorporates horizons by the topological "no boundary" condition. Here we have shown that the smoothness of the euclidean black hole manifolds at the horizon is ultimately responsible for the continuous spectrum in lorentzian correlation functions.

Thus, the horizon "no boundary" condition is *thermostatic* in nature and fundamentally "coarse grained", missing important information about the details of the thermal ensemble (see also [29]). We have seen that the recurrences can be restored by a singular modification of the smooth manifold X, such as the brick-wall boundary condition of [15]. A more physical boundary condition at the horizon is only obtained within a nonperturbative formulation of quantum gravity, such as the AdS/CFT correspondence. In this respect, the infinite storage capacity of states at the horizon violates the "stringy exclusion principle" [24].

One limitation of these ideas is the specific nature of concrete AdS/CFT dual pairs, since the correspondence provides a quantum interpretation of large black holes *together* with the asymptotic AdS space. It is much less clear how to generalize these considerations to large black holes in general spacetimes. Even the CFT state that corresponds to small, unstable black holes in AdS has not been identified.

One point of view would be that quantum properties of a black hole can be specified by a sort of "quantum horizon", independently of the particular asymptotic vacuum where the black hole sits. One can imagine that horizons support a subspace $\mathcal{H}_H$ of the total Hilbert space of states, and that the structure of $\mathcal{H}_H$ has some universal features. This is one of the implicit assumptions of the so-called "stretched horizon" phenomenological model [30, 31, 32], as well as approaches that involve dynamical limits with change of the vacuum (c.f. for example [33, 34]). While semiclassically all stretched horizons are described as thermodynamical "hot surfaces" with internal entropy ('t Hooft's brick-wall model being

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just the crudest version of the stretched horizon), the AdS/CFT correspondence gives no concrete evidence that this universality should extend to the detailed quantum structure. In fact, the confusing situation with the quantum description of de Sitter space would speak to the contrary.

Nevertheless, it is possible that part of the quantum structure of stretched horizons is accesible within a systematic semiclassical expansion in the bulk theory. Time averages of correlators are natural candidates to test this idea, being of order $\exp(-1/\lambda^2)$, where $\lambda^2 \sim G_{\rm N}$ is the effective expansion parameter of the gravitational theory (string, M-theory or CFT dual). This suggests that the recurrences could be restored by summing appropriate instantons. We have examined one class of instanton transitions proposed in [2], in terms of large-scale topology-changing fluctuations of the geometry. Although the expected time-averaged bound is obtained in order of magnitude, there are reasons to believe that the instanton approximation fails to capture the Poincaré recurrences. Specifically, both the time scale of correlator "bumps" and their amplitude are much smaller than expected. While this class of instanton corrections gives an interesting long-time noise to the correlation functions, foretelling the restoration of unitarity, the large recurrences still stay out of reach of these particular semiclassical processes.

These considerations show the inherent limitations of the "master field" approximation when it comes to reveal fine details of the quantum mechanical system. Incidentally, this might be important in related contexts, such as the program of extracting information about spacelike singularities from thermal CFT correlators, c.f. [28, 35].

It would be useful to have a general picture of how recurrences are to be retrieved from a given microscopic model of the stretched horizon, independently of the particular large-distance asymptotics of the whole system. Our results indicate that large-scale instanton contributions are not enough. Perhaps instanton effects on shorter scales can be identified as the main actors. In this respect, it is worth pointing out that the smoothness of the euclidean black-hole saddle point is the result of a fine-tunning between the curvature and the temperature identification. If back-reaction effects induce temperature fluctuations this translates into conical singularities in X. In string theory this could bring interesting dynamics of closed-string tachyon condensation (c.f. [36, 37, 27, 38, 39]).

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